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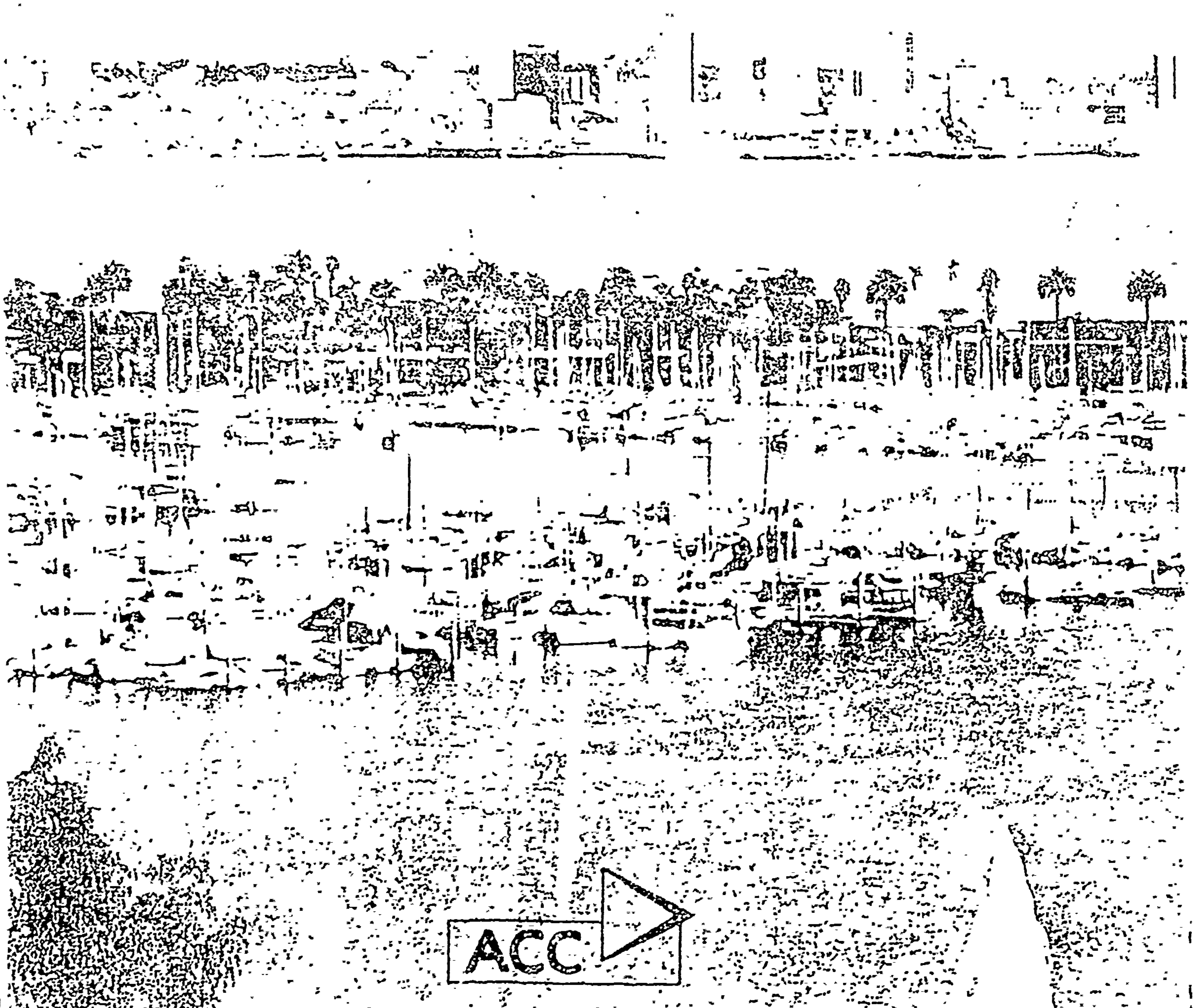

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Volume 2 of 3

AN APPROACH TO THE TARGET DATA ASSOCIATION PROBLEM USING SUBJECTIVE AND STATISTICAL INFORMATION

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Abstract

A general approach is presented to the data association problem in tracking which incorporates both linguistic-based and statistical information, extending classical results. The technique continues the previous effort of the PACT (Possibilistic Approach to Correlation and Tracking) algorithm. In addition, asymptotic consistency results are obtained relative to increasing sample sizes and model refinements.

1. Introduction

The PACT algorithm has been documented in previous papers, [1],[2],[3]. It is an attempt to perform track-to-track or report-to-track data association or "correlation" based upon varying information sources. The procedure in an obvious way may be extended to problems of medical diagnosis, fault determination, and classification [1]. The entire technique is based upon the idea that attributes may be represented by fuzzy sets and that such objects represent a weakening of probabilistic descriptions-and hence suitable for the interpretation of natural language information([3];[4] for other work).

2. Hybrid Logical Systems

A hybrid logical system consists of the tuple $F=(\mathcal{O}_{not},\mathcal{O}_{\&},\mathcal{O}_{or};\mathcal{U})$ where: $\mathcal{O}_{not}$ is a collection of negation operators $\phi_n:[0,1]\to[0,1]$ nonincreasing with $\phi_n(0)=1,\phi_n(1)=0$. Other properties that ϕ_n can possess include involution and continuity [5]. $\mathcal{O}_{\&}$ is a collection of conjunction or general cartesian product operators $\phi_{\&}:[0,1]^2\to[0,1]$ nondecreasing, continuous, bounded above by min, and such that for all u,v: $\phi_{\&}(0,v)=\phi_{\&}(u,0)=0$; $\phi_{\&}(1,v)=v=\phi_{\&}(v,1)$, (1) as well as higher argument operators with similar properties. When $\phi_{\&}$ as a two place operator is symmetric and associative, then it is called a t-norm, and possesses many useful properties, including an unambiguous recursive extension to multiple arguments and canonical representations [5],[6]. When n-argument $\phi_{\&}$ is the cumulative distribution function (c.d.f.) of uniform [0,1] random variables (r.v.) (not necessarily independent, nor $\phi_{\&}$ need be symmetric or associative), it is called a copula [6]. Copulas and t-norms may coincide or not. (See [7].) A similar situation holds dually for $\mathcal{O}_{or}$, a collection of disjunction or general cartesian sum operators $\phi_{or}:[0,1]^m\to[0,1]$, for various m, nondecreasing continuous, bounded below by max, and e.g., for m=2 all u,v: $\phi_o(1,v)=\phi_o(u,1)=1$; $\phi_o(0,v)=v=\phi_o(v,0)$. (2) t-conorms and co-copulas are also defined analogously [5],[6]. Finally, $\mathcal{U}$ is a collection of fuzzy set membership or possibility functions $\phi_A:X\to[0,1]$, corresponding uniquely to attribute or fuzzy subset A of space X having domain $dom(A)=support(\phi_A)\subseteq X$; for various A and X. Such functions include ordinary probability functions when X is discrete, ordinary cdf's when $X\subseteq\mathbb{R}^n$, unimodal normable continuous or not (normable=having unit value somewhere), etc. (See, e.g [8] for various basic properties of fuzzy sets and possibility functions as well as many applications.)

The collection S of all strings produced by F consists of all well-defined combinations of operators from F evaluated at arbitrary possibility functions chosen from $\mathcal{U}$ and possibly further evaluated at functional arguments. A typical string is denoted

$$comb(..\phi_n..;..\phi_{\&}..;..\phi_o..;..\phi_A..)(...,x,...) \quad (3)$$

The following principles involving linguistic/semantic evaluations are assumed :

Principle 1 - Parsing

All statements in natural language may be expressed in a formal language in terms of memberships or degrees of compatibility of elements w.r.t. attributes and the use of operators forming formal strings representing (possible different) not, &, or. Symbolically, a typical string here is

$$s \overset{d}{=} comb(..not..;..\&..;..or..;..(x\varepsilon A)..) \quad (4)$$
$$= (..,x,..)\varepsilon(comb(..not..;..\&..;..or..)(..A..));$$

Principle 2 - Abstraction

The truth or semantic content of any string (or proposition) as in eq.(4) is tr(s), as in eq.(3). If the system is truth functional, then (by definition)

$$tr(s)=comb(..\phi_n..;..\phi_{\&}..;..\phi_o..)(...,\phi_A(x),...) \quad (5)$$

Other concepts associated with natural language may be described and evaluated, including: conditioning, quantification, interaction/independence as well as dependence, projections and marginal attributes; tense, mode, case and prepositional and predicative relations; and functional and relational transforms, including arithmetic operations and linguistic modifications. (See for example, [4] or [9].) Examples of the above will be given later in association with the PACT algorithm essentially as error distribution and inference rules.

One example of a hybrid system is simple probability logic where $F=(neg;\phi_{\&};\phi_o;\mathcal{U}_{cdf})$, where $neg(x)=1-x$; $\phi_{\&}$ is a fixed n (argument) copula, ϕ_o is the DeMorgan transform of $\phi_{\&}$ and thus a co-copula, and $\mathcal{U}_{cdf}$ is the collection of all cdf's over $\mathbb{R}$. Then by Sklar's Theorem [7] for any $F_1,..F_n$ $\phi_{\&}(F_1,...,F_n)$ (in compositional form) is a simple string representing a legitimate cdf. By replacing F_j's by 1, marginal cdf's or cdf's of number less

than n can be formed. More generally, non-cdf string

$$\mathrm{comb}(\mathrm{neg};\psi_{\&},\psi_0)(F_1,..F_m)(x_1,..x_m) \tag{6}$$
$$= \psi_{\&}(F_1,...,F_m)(\mathrm{comb}(C,X,+)((-\infty,x_1],...,(-\infty,x_m])),$$

which may be extended uniquely above the ray level to arbitrary Borel sets, but for the latter situation, F is no longer truth functional in general (a characteristic of jointness of events in probability- see, e.g. Rescher [10]).

More generally, a Lebesgue-Stieltjes logic may be developed, where cdf's are replaced by functions of bounded variation and probability measures by signed measures at the Borel set level [9]. Of course, in the probability logic example, the single $\psi_{\&}$ could be replaced by a collection of copulas and related functions (noting that in general copulas composed with multiple copulas are not copulas, although they do remain conjunction operators).

A single natural language concept may lead to a variety of possible semantic evaluations. One contributing factor to this ambiguity lies in the nonuniqueness of extending an ordinary function $f:X_1\times X_2\to Y$ to $\bar{f}:\mathcal{F}(X_1)\times\mathcal{F}(X_2)\to\mathcal{F}(Y)$, the lifting of f, where $\mathcal{F}(X)$ denotes an appropriate class of attributes of X (or all attributes on X). Two candidates for $\bar{f}$ are, using Principles 1 and 2 and the unique representation of attributes through their membership functions for a simple logical system:

$$\phi_{\bar{f}(A,B)}(y)=tr(y\varepsilon f(A,B))=tr((\exists(x,z)\varepsilon A\times B)\&(y=f(x,z))) = \underset{\left(\text{all } (x,z)\varepsilon f^{-1}(y)\right)}{\psi_0}\left(\psi_{\&}(\phi_A(x),\phi_B(z))\right) \tag{7}$$

and

$$\phi_{\bar{f}(A,B)}(y)=tr(f^{-1}(y)\varepsilon A\times B)=\psi_{\&}(\phi_A,\phi_B)(f^{-1}(y)), \tag{8}$$

where in the first case, $A\varepsilon\mathcal{F}(X_1)$, $B\varepsilon\mathcal{F}(X_2)$, and in the second case, $A\varepsilon\mathcal{F}(P(X_1))$, $B\varepsilon\mathcal{F}(P(X_2))$, related to the measure theory concept of transformation of measure.

In any case, once the functional extension problem is determined, other ambiguities arise: Let $\mathcal{BP}$ be the class of all multiple and single argument Boolean truth polynomials, i.e.,

$$\mathcal{BP} = \bigcup_{n=1}^{+\infty}\left(\{0,1\}^{\{0,1\}^n}\right). \tag{9}$$

Thus each function f in $\mathcal{BP}$ may be extended to $\bar{f}$: $\mathcal{F}[0,1]\times..\times\mathcal{F}[0,1]\to\mathcal{F}[0,1]$. Call this class $\bar{\mathcal{BP}}$. Now, $f=\mathrm{comb}(\psi_n;\psi_{\&};\psi_0)$ restricted to $\{0,1\}^m$ for suitable choices of m yields a classical logical connective , and in turn the extension $\bar{f}$ thus provides an alternative natural evaluation to eq.(5)

$$tr(s)=(\phi_{\bar{f}(..(A,CA)..)}(0),\phi_{\bar{f}(..(A,CA)..)}(1)), \tag{10}$$

in general, distinct in form from that in eq.(5).

Another problem is the choice of the operators for F to model a given situation. Restricting these operators to be , e.g., t-norms or copulas may be confining, despite the appeal. This is closely related to Hilbert's 13th problem, and more generally, to the problem of representing multiple argument functions by superpositions of lower argument functions [11], when associative copulas or t-norms are employed, because of their canonical representations. On the positive side, it can be shown that Sprecher's refinement of Kolmogorov's original form for an arbitrary continuous function of multiple arguments may be formally couched as the string for constants c_j,

$$\underset{j=1,..,n}{\mathrm{bndsum}}(g(\psi_{\&}(\psi_{\&}^{(\lambda_1)}(\phi_{A_1}(x_1)),...,\psi_{\&}^{(\lambda_n)}(\phi_{A_n}(x_n)),c_j))), \tag{11}$$

where bndsum is ordinary sum bounded by unity, a t-conorm, g is monotone decreasing , and $\psi_{\&}$ is a strict (and hence Archimedean) t-norm having canonical representation

$$\psi_{\&}(x_1,..x_n)=h^{-1}(h(x_1)+..+h(x_n)), \tag{12}$$

where $h:[0,1]\to\mathbb{R}^+$ is continuous decreasing with $h(0)=+\infty$ and $h(1)=0$. Also, the λ-intensification or modification of A through $\psi_{\&}$ is given by

$$\psi_{\&}^{(\lambda)}(\phi_A(x))\overset{d}{=}h^{-1}(\lambda h(\phi_A(x))), \tag{13}$$

noting that these intensifications obey the exponential law

$$(\psi_{\&}^{(\lambda')})^{(\lambda'')}=(\psi_{\&}^{(\lambda'')})^{(\lambda')}=\psi_{\&}^{(\lambda'\lambda'')} \tag{14}$$

for all nonnegative real λ, extending the integer-valued repetition of operator $\psi_{\&}$. A reasonable substitution for the above form is the use of ordinary exponentiation or translation of arguments. (See [3])

3. Combination of Evidence

Let F be a given hybrid logical system to be used for the relevant modeling. Let $\theta\in X$ be an unknown parameter vector to be estimated or decided upon. Suppose that $\phi_{A_j}:X\times Y_j\to[0,1]$, $j=1,..,m$, are m possibility functions describing θ as well as introducing nuisance parameters $Z_j\,\varepsilon Y_j$, $j=1,..,m$.

Then , if for example a single m-copula $\psi_{\&}$ is chosen for combination, one has the joint non-interactive function

$$\phi_A(\theta,Z)=\psi_{\&}(\phi_{A_j}(\theta_j,Z_j)) , \quad (j=1,..,m) \tag{15}$$

$$\theta\overset{d}{=}(\theta_1,...,\theta_m) \;;\; Z\overset{d}{=}(Z_1,...,Z_m) . \tag{16}$$

This representation of the joint information and all nuisance values may be interpreted in terms of either ordinary operations upon nested level sets or equivalently upon cartesian products ; in particular, on nested random sets. (Indeed, a strong isomorphic-like relation holds between many common set operations on nested(random)sets and corresponding possibility functions, including intersections, unions, set differences, projections, and functional transforms. See [6] and [12].) Thus, for all θ,Z ,

$$\phi_A(\theta,Z)=Pr((\theta_1,Z_1)\varepsilon S_{U_1}(A_1)\&..\&(\theta_m,Z_m)\varepsilon S_{U_m}(A_m))) \tag{17}$$

where $S_{U_j}(A_j)\overset{d}{=}\phi_{A_j}^{-1}[U_j,1]$ is the unique nested random subset of $X\times Y_j$ which is one point coverage equivalent to ϕ_{A_j} , where $U_1,..U_m$ are marginally rv's distributed uniformly over $[0,1]$ with joint cdf $\psi_{\&}$. (See [6],[7] for other properties and results.)

Next, consider the concept of conditional possibility functions: Let X and Y be given spaces with attribute $B\in\mathcal{F}(X\times Y)$ dominated by attribute $C\in\mathcal{F}(Y)$ i.e.,

$$\sup_{(x\varepsilon X)}\phi_B(x,y)\le\phi_C(y) \;;\; \text{all } y\varepsilon Y . \tag{18}$$

Then it follows that there exists (uniquely, if, eg, $\psi_{\&}$ is a strict t-norm) conditional attributes $(B|y)$ satisfying (for given operator $\psi_{\&}$) for all $x\varepsilon X$, $y\varepsilon Y$

$$\phi_B(x,y)=\psi_{\&}(\phi_{(B|y)}(x),\phi_C(y)) . \tag{19}$$

In particular, let the diagonal possibility descrip-

tion of θ be , assuming here a simple system,

$$\phi_{d(A)}(\theta)^{d} \; tr(\theta_1 = \dots = \theta_m = \theta \; \epsilon \bigcap_{1\le j\le m} proj_x(A_j))$$

$$= \phi_{\underset{(all\ Z)}{o}} (\phi_A(\theta,\dots,\theta;Z)). \qquad (20)$$

Let $X = X$ and Y be the space of all possible descriptions (or "possible worlds") of θ. Hence, $d(A)\epsilon Y$ and the universal inclusion description $D\epsilon F(Y)$ with

$$\phi_D(d(A)) = tr((\exists\theta)(\theta\epsilon d(A))) = \phi_{\underset{(\theta\epsilon X)}{o}} (\phi_{d(A)}(\theta)). \qquad (21)$$

Since the compound attribute representing θ and $d(A)$ has the same evaluation, it follows that

$$\phi_{d(A)}(\theta) = \phi_{\&}(\phi(\theta|d(A)), \phi_D(d(A))) \qquad (22)$$

determines the final combination of evidence function $\phi(\theta|d(A))$ as a function of $\theta\epsilon X$.

As a check on this procedure reducing to the classical combination of statistical evidence, if $\phi_{\&}$=prod, ϕ_{or}=bndsum, Y_j is vacuous, and $X=R^n$ suitably discretized where each ϕ_{A_j} is approximated by

$$\phi_{A_j}(\theta_j) = \varphi(\theta_j - u_j, \Lambda_j)\cdot\Delta(\theta_j) \;, \qquad (23)$$

φ being the standard probability density function (p.d.f.) for the gaussian distribution $N(0,I)$, then

$$\phi(\theta|d(A)) = \varphi(\theta - \bar{u}, \bar{\Lambda})\cdot\Delta(\theta) \;, \qquad (24)$$

$$\bar{u} \overset{d}{=} \bar{\Lambda}\cdot\sum_{j=1}^{m}\Lambda_j^{-1}\cdot u_j \;;\; \bar{\Lambda} \overset{d}{=} (\sum_{j=1}^{m}\Lambda_j^{-1})^{-1} \;, \qquad (25)$$

the same formally as the BLUE (best linear unbiased estimator) of u, and by a fiducial argument, of θ.

4. Application to Data Association

Suppose now that attributes $A_1,\dots,A_m$ describing nuisance parameters concerned with θ^m (but not θ directly) are available in the form of compound attributes represented by error functions and by inference rules. A_1 could represent geolocation, A_2 a sensor system state, A_3, another state, and for $m=4$, A_4 could represent visual descriptions, where θ represents the level of correlation between a pair of track histories, say i and j. Thus, Z is described separately by error functions $\phi_{P_k}(Z_k^{(\ell)}|\tilde{Z}_k^{(\ell)})$ for $\ell=i,j$ and $k=1,\dots,m$, where $\tilde{Z}_k^{(\ell)}$ are observed or predicted outcomes of data associated with A_k - in fact, $\tilde{Z}_k^{(\ell)} \;\epsilon\; dom(A_k)$ - and $Z_k^{(\ell)} \;\epsilon\; dom(A_k)$ can be arbitrary. Denote

$$Z=(Z_1,\dots,Z_m) \;\;;\; Z_k=(Z_k^{(i)},Z_k^{(j)}) \;, k=1,\dots,n \qquad (26)$$

with similar notation for $\tilde{Z}$ and $\tilde{Z}_k$.

In addition, Z and θ are described jointly(or perhaps, more accurately, conditionally) by r inference rules R_t, $t=1,\dots,r$, where

$$\phi_{R_t}(\theta|^t Z) = tr((Z \;\epsilon\; G_t) \text{ iff}(\theta \;\epsilon\; Corr_{\gamma_t}))$$

$$= \phi_{\rightarrow}(\phi_{G_t}(Z), \phi_{Corr_{\gamma_t}}(\theta)) \;, \qquad (27)$$

where implication operator $\phi_{\rightarrow} = \phi_{\Rightarrow}$ or $\phi_{\Leftrightarrow}$, corresponding to "if-then" and "iff", respectively. For example, one can define for all $u,v\epsilon[0,1]$, (28)

$$\phi_{\Leftrightarrow}(u,v) = \phi_{\&}(\phi_{\Rightarrow}(u,v), \phi_{\Rightarrow}(v,u)) ; \phi_{\Rightarrow}(u,v) = \phi_o(\phi_n(u), v)$$

where antecedent

$$\phi_{G_t}(Z) = tr(\underset{(k\epsilon J_t)}{\&} (Z_k^{(i)} \;\&\; Z_k^{(j)} \text{ match to degree } \beta_{t,k}))$$

$$= \underset{(k\epsilon J_t)}{\phi_{\&}} (\phi_{F_{\beta_{t,k}}}(\phi_{M_k}(Z_k))) \;, \qquad (28)$$

$J_t \subseteq \{1,\dots,m\}$ is index set of attributes R_t operates on, M_k is a matching 'set' for attribute A_k so that $\phi_{M_k}(Z_k)$ is the degree to which $Z_k^{(i)}$ and $Z_k^{(j)}$ are compatible, with intensification (or extensification) determined by $\phi_{F_{\beta_{t,k}}}$ for some $\beta_{t,k}\ge 0$, either in the form as given in eq.(13) or (as previously mentioned) as exponential or translational argument form. $Corr_{\gamma_t}$ similarly represents γ_t-degree correlation. A table can be established relating linguistic intensifications with numerical ones-$\beta_{t,k}$, γ_t.

From now on, simplifying notation for all relevant error and matching functions and inference rules will be employed. The basic problem here is to apply the combination of evidence procedure to the data association problem for an appropriately chosen hybrid logical system. First note that:

A. Attributes may be grouped roughly into two parts: $\{A_1,\dots,A_{m'}\}$, statistical attributes ; $\{A_{m'+1},\dots,A_m\}$, subjective attributes. For each statistical attribute A_k such as geolocation or sensor state parameter, P_k arises typically as the discretization of pdf f_k, say, obtained by various geometric and physical considerations. For each subjective attribute A_k such as identification or visual description of some type, P_k is obtained from a panel of experts and represents possibility values conditioned on potential observed data and in general will be approximately a symmetric binary function, so that a reasonable choice for M_k in any R_t is $M_k(Z_k)=P_k(Z_k^{(i)}|Z_k^{(j)})$.

B. On the other hand, for any statistical attribute A_k, in general M_k is not the same as P_k, since statistical hypotheses testing (involving P_k) presents a natural candidate for M_k : the significance level function for one-dimensional test statistic n_k, say, where $n_k=n_k(\tilde{Z}_k)$ for any outcome of rv $\tilde{Z}_k$, noting again that P_k is a discretization of f_k the pdf of $(\tilde{Z}_k^{(\ell)}|Z_k^{(\ell)})$, $\ell=i,j$. Thus, for the hypotheses H_0: i,j match vs. H_1: i,j don't match , one obtains

$$M_k(Z_k) = Pr(n_k(\tilde{Z}_k) > n_k(Z_k)|H_0) = 1 - T_k(n_k(Z_k)), \qquad (29)$$

where T_k is the cdf of $(n_k(\tilde{Z}_k)|H_0)$.

C. Inference rules are generally selected from a panel of experts and are thus uniquely determined by -once a choice of F is made- J_t and the tuple $((\beta_{t,k})_{k\epsilon J_t};\gamma_t)$, although modifications to this may occur if negative matching or negative correlation relations are introduced.

The following three theorems will be useful in developing principles for choosing appropriate classes of operators for F.

Theorem 1.

Let $\theta\epsilon X$ with $B_1, B_2,\dots \;\epsilon F(X)$, $\phi_{\&}$ a conjunction operator, and form for any possible value x of θ,

$$\phi_B(x) = \phi_{\underset{k=1,2,\dots}{\bigcap} B_k}(x) = \phi_{\&}(\phi_{B_1}(x), \phi_{B_2}(x),\dots). \qquad (30)$$

Then:

(i) If $\phi_{\&}$ is either an Archimedean t-norm or min and

$$\{x|x\epsilon X \;\&\; \lim_{k\to\infty} \phi_{B_k}(x)=1\} = \emptyset \;, \qquad (31)$$

then, unless $\phi_{\&}$ = min, $\phi_B(x) = 0$, for all $x\epsilon X$.

(ii) The α-level description of θ by level sets equivalent to $B_1, B_2, \ldots$ is

$$g(\alpha) = \bigcap_{1\le k<+\infty} (\phi_{B_k}^{-1}[\alpha,1]) \subseteq X \;; 0\le\alpha\le 1. \quad (32)$$

Then

$$\phi_B^{-1}[\alpha,1] \subseteq g(\alpha) \;,\; \text{all } 0\le\alpha\le 1 \;, \quad (33)$$

with proper subset relation holding in general, and with equality holding iff $\phi_\&$ = min, i.e., the only case where no information is lost concerning θ is min.

Theorem 2.

Let $C\in P(R^n)$ with the p^{th} discretization $C_p \in F(D_p)$ of C defined by letting D_p be the domain of C_p formed by making a p^{th} discretization of R^n with mesh fineness strictly decreasing as p grows, etc., and where

$$\phi_{C_p}(x) = \phi_C(x) \;; \text{all } x\in D_p \;. \quad (34)$$

Then,

(i) If ψ_0 is an Archimedean t-conorm,

$$\lim_{p\to+\infty} \psi_0 \underset{(x\in D_p)}{(\phi_{C_p}(x))} = 1 \;. \quad (35)$$

(ii) If ψ_0 = max,

$$\lim_{p\to+\infty} \psi_0(\phi_{C_p}(x)) \le 1 \;, \quad (36)$$

with, in general, strict inequality holding.

Theorem 3. (See also [13].)

(i) Let $f_k: R^{m_k} \to R^+$, $k=1,\ldots,a$, for a, fixed finite, be bounded pdf's.

(ii) Let $\psi_{\&1}$ be a conjunction. Define the p^{th} modified discretization $f_{k,p}$ of f_k as

$$f_{k,p}(x_k) \overset{d}{=} f_k(x_k)\cdot\Delta(x_k) \;; x_k \in D_{k,p} \;, \quad (37)$$

where $D_{k,p}$ is the corresponding discrete domain and Δ is the mesh function so that

$$\sum_{(x_k\in D_{k,p})} f_{k,p}(x_k) = 1 \;; \quad (38)$$

and finally, let for any $x=(x_1,\ldots,x_a) \in D_{(p)} \overset{d}{=} D_{1,p}\times\cdots\times D_{a,p}$,

$$f_{(p)}(x) \overset{d}{=} \psi_{\&1}(f_{1,p}(x_1),\ldots,f_{a,p}(x_a)). \quad (39)$$

(iii) Suppose also that

$$h_a \overset{d}{=} (\partial^a \psi_{\&1}(u_1,\ldots,u_a)/\partial u_1\cdots\partial u_a)_{u_1=\cdots=u_a=0} \quad (40)$$

exists finitely.

(iv) Suppose that $\psi_{\&2}$ is a conjunction such that there is δ, 0<δ<1, such that for all $u\in[0,1]$, $\partial\psi_{\&2}(u,v)/\partial v$ and $\partial^2\psi_{\&2}(u,v)/\partial^2 v$ are bounded for all $v\in[0,\delta)$.

(v) $\phi_\Gamma: X\times R^{m_0} \to [0,1]$ is a given continuous function where X is the space that contains θ and where

$$m_0 \overset{d}{=} m_1+\cdots+m_a \;. \quad (41)$$

(vi) It is assumed that ψ_0 is an Archimedean t-conorm with canonical generator h: h is continuous with

$$\psi_0(u,v) = 1-h^{-1}(\min(h(1-u)+h(1-v), h(0))) \;, \quad (42)$$

for all $u,v\in[0,1]$, and $h:[0,1]\to R^+$ is nonincreasing with $h(1)=0$. (See [5] for further properties.) It is further assumed that there is a κ, 0<κ<1, such that $dh(v)/dv$ and $d^2h(v)/d^2v$ are bounded functions of $v\in[\kappa,1]$.

Then, for all $\theta\in X$,

$$\lim_{p\to+\infty}(\underset{(x\in D_{(p)})}{\psi_0} (\psi_{\&2}(\phi_\Gamma(\theta,x), f_{(p)}(x))))$$

$$= 1-h^{-1}(\min(\tau(\theta), h(0))) \;, \quad (43)$$

where

$$\tau(\theta) \overset{d}{=} h_a \cdot \int_{x\in R^{m_0}} \left(k(\phi_\Gamma(\theta,x))\cdot\prod_{k=1}^{a} f_k(x_k)dx_k\right), \quad (44)$$

$$k(u) \overset{d}{=} -(dh(v)/dv)_{v=1}\cdot(\partial\psi_{\&2}(u,v)/\partial v)_{v=0} \;, \quad (45)$$

for all $u\in[0,1]$.

Remark.

Frank's family of modular t-norms (see [5] or [6] for various properties) and t-conorms can be shown to satisfy the conditions of Theorem 3 for any $\psi_{\&1}$, $\psi_{\&2}$ and ψ_0 chosen from the Archimedean part of the family.

Theorems 1,2,3 lead to the following:

Principle 3

If $\phi_{B_k}: X\to[0,1]$, k=1,2,.. is a sequence of descriptions of $\theta\in X$ with either high redundancy or much irregularity occurs in the sequence, $\lim_{p\to+\infty} \phi_{B_k}(\theta) = 1$ does not appear viable, t[illegible] for combining this sequence $\psi_\&$=min. In addition, min allows for an uncountable number of descriptions of θ.

Principle 4

If ϕ_{B_k}'s as above are now relatively non-interactive/non-overlapping, then $\psi_\&$ and ψ_0 as Archimedean t-norms and t-conorms, respectively, may be reasonable.

Principle 5

If C_p is the p^{th} discretization of C as in Theorem 2, then for the model, choose ψ_0=max so that $\lim_{p\to+\infty} (\underset{x\in D_p}{\psi_0}(\phi_{C_p}(x))$ becomes non-trivial.

Thus, returning to the basic problem, Principle 4 implies that if $A_1,\ldots,A_{m'}$ are all relatively non-overlapping, then the total error effect may be obtained by simply choosing an appropriate conjunction $\psi_{\&1}$ which may well be an Archimedean t-norm such as from Frank's family. This yields the overall statistical error effect

$$P(Z'|\mathring{Z}') \overset{d}{=} \underset{(1\le k\le m')}{\psi_{\&,1}}(P_k(Z_k|\mathring{Z}_k)) \;, \quad (46)$$

where

$$P_k(Z_k|\mathring{Z}_k) \overset{d}{=} (P_k(Z_k^{(1)}|\mathring{Z}_k^{(1)}), P_k(Z_k^{(j)}|\mathring{Z}_k^{(j)})) \quad (47)$$

for all k, k=1,...,m.

Similarly, for $A_{m'+1},\ldots A_m$, the overall subjective error effect is

$$P(Z''|\mathring{Z}'') \overset{d}{=} \underset{(1\le k\le m)}{\psi_{\&,2}}(P_k(Z_k|\mathring{Z}_k)) \;, \quad (48)$$

with possibly the two conjunctions being chosen the same, and further, as an Archimedean t-norm.

On the other hand, since inference rules typically operate on combinations of common or similar attributes - by intensification modifications- Prin-

ciple 3 may be invoked to yield the overall inference rule effect as

$$R(\theta|Z)=\min_{1\le t\le r}(R_t(\theta|Z)) . \qquad (49)$$

Again using Principle 4, it may be appropriate to choose Archimedean t-norm $\psi_{\&3}$ to obtain the joint posterior function

$$\phi(\theta,Z|\overset{o}{Z},R,P)\overset{d}{=}\psi_{\&3}(P(Z|\overset{o}{Z}),R(\theta|Z))$$
$$=\psi_{\&3}(P(Z'|\overset{o}{Z}'),\phi_r(\theta,Z)) , \qquad (50)$$

where overall error effect is

$$P(Z|\overset{o}{Z})\overset{d}{=}\psi_{\&3}(P(Z'|\overset{o}{Z}'),P(Z''|\overset{o}{Z}'')) \qquad (51)$$

and, as in Theorem 3,

$$\phi_r(\theta,Z)\overset{d}{=}\psi_{\&3}(P(Z''|\overset{o}{Z}''),R(\theta|Z)). \qquad (52)$$

Note that a sufficiency condition holds here for

$$\phi(\theta|Z,\overset{o}{Z},R,P)=\phi(\theta|Z,R)=R(\theta|Z) \qquad (53)$$

as well as for

$$\phi(Z|\overset{o}{Z},R,P)=\phi(Z|\overset{o}{Z},P)=P(Z|\overset{o}{Z}), \qquad (54)$$

which also when substituted into (50) yields a form which may be independently derived by use of conditioning of possibility functions.

Next, Principle 5 may be used to obtain the marginal posterior function for θ, before normalization, using the universal inclusion description:

$$\phi_{d(A)}(\theta)=\phi(\theta|\overset{o}{Z},R,P)=\max_{\text{all } Z''}(\max_{\text{all } Z'}\psi_o(\phi(\theta,Z|\overset{o}{Z},R,P))), \qquad (55)$$

for some choice of Archimedean t-conorm ψ_o, followed in turn by again invoking Principle 5,

$$\phi_D(d(A))=\sup_{0\le\theta\le 1}(\phi(\theta|\overset{o}{Z},R,P)) , \qquad (56)$$

which finally yields the normalized posterior $\phi(\theta|d(A))$ (dependent upon $\overset{o}{Z},R,P$), by use of (22).

Note that if $P_k(Z_k|\overset{o}{Z}_k)=\delta_{Z_k,\overset{o}{Z}_k}$ (Krönecker delta), then the evaluation of all R_t involving that attribute A_k, in effect have $M_k(Z_k)$ replaced by $M_k(\overset{o}{Z}_k)$ in the antecedent and the term $P_k(Z_k|\overset{o}{Z}_k)$ is dropped from the calculations for the (normalized) posterior. In the above computations, for simplicity, the $\psi_{\&\ell}$ for $\ell=1,2,3$ could be set equal, if other conditions do not arise.

5. Asymptotic Behavior

Suppose that the combination of evidence problem relative to data association is carried out as outlined in the previous section, where P_k is the p^{th} discretization (modified) of pdf f_k, for $k=1,\ldots,m'$, and all corresponding conditions for parts (i)-(iv) and (vi) for Theorem 3 hold ($a=2m'$). Also, note that part (v) of Theorem 3 is satisfied by use of eq.(52) and that any given inference rule R_t may use implication operator $\to$ being either $\Rightarrow$ or $\Leftrightarrow$.

(A) Under the above general conditions, for all $\theta\in[0,1]$,

$$\lim_{p\to+\infty}(\phi(\theta|\overset{o}{Z},R,P))=1-h^{-1}(\min(\tau(\theta),h(0))) \qquad (57)$$

as in eq.(43), but changed to account for max, where

$$\tau(\theta)=h_{2m'}\cdot\max_{\text{all } Z''}(E_V(k(\phi_r(\theta,(V,Z''))))) , \qquad (58)$$

where Z in eq.(52) is replaced by (V,Z''), V being a r.v. representing formally Z' and having pdf f':

$$f'(Z'|\overset{o}{Z}')=\prod_{k=1}^{m'}(f_k(Z_k^{(i)}|\overset{o}{Z}_k^{(i)})\cdot f_k(Z_k^{(j)}|\overset{o}{Z}_k^{(j)})). \qquad (59)$$

(B) If also both statistical and possibilistic consistency holds, i.e.,

$$f_k(Z_k^{(\ell)}|\overset{o}{Z}_k^{(\ell)})\to\delta(Z_k^{(\ell)}-\overset{o}{Z}_k^{(\ell)}) \text{ (dirac delta)} \qquad (60)$$

for $k=1,\ldots,m'$, and

$$P_k(Z_k^{(\ell)}|\overset{o}{Z}_k^{(\ell)})\to\delta_{Z_k^{(\ell)},\overset{o}{Z}_k^{(\ell)}} \text{ (Krönecker dlta)} \qquad (61)$$

for $k=m'+1,\ldots,m$, and if k is such that k is nondecreasing with $k(0)=0$ (as is true for the members of Frank's family-see [9]), then

$$\tau(\theta)=h_{2m'}\cdot k(R(\theta|\overset{o}{Z})) \qquad (62)$$

and hence

$$\lim_{p\to+\infty}(\phi(\theta|\overset{o}{Z},R,P))\le 1-h^{-1}(h_{2m'}\cdot k(1)). \qquad (63)$$

(C) Suppose the conditions in (A) and (B) obtain. Then

$$R(\theta|\overset{o}{Z})\le\min_{1\le t\le r}(\psi_o(1-G_t(\overset{o}{Z}),\theta^{\gamma_t}))$$
$$\le\min_{1\le t\le r}(\operatorname{card}(J_t)-\sum_{(k\in J_t)}M_k(\overset{o}{Z}_k)^{\beta_{t,k}}+\theta^{\gamma_t})$$
$$\overset{d}{=}\upsilon_o . \qquad (64)$$

(D) Thus, if also for $k=1,\ldots,m$,

$$1\ge M_k(\overset{o}{Z}_k)\ge 1-\omega \ ; \ 0\le\omega<1 \text{ constant}, \qquad (65)$$

then

$$R(\theta|\overset{o}{Z})\le\min_{1\le t\le r}(\operatorname{card}(J_t)(1-(1-\omega)^{\beta_t})+\theta^{\gamma_t})$$
$$\le m\cdot(1-(1-\omega)^{\beta_o})+\theta^{\gamma_o}$$
$$\overset{d}{=}\varepsilon_o , \qquad (66)$$

where

$$\beta_t\overset{d}{=}\max_{(k\in J_t)}(\beta_{t,k}) ; \ \beta_o\overset{d}{=}\max_{(1\le t\le r)}(\beta_t);\gamma_o\overset{d}{=}\max_{(1\le t\le r)}(\gamma_t) . \qquad (67)$$

Hence, for all $\theta\in[0,1]$,

$$\lim_{p\to+\infty}(\phi(\theta|\overset{o}{Z},R,P))\le 1-h^{-1}(h_{2m'}\cdot k(\varepsilon_o)). \qquad (68)$$

Thus, if also $M_k(\overset{o}{Z}_k)\to 1$ uniformly, for $k=1,\ldots,m$, β_o is bounded positively below, and γ_o increases without bound, then it follows from (66) and (68) that for all $0\le\theta<1$,

$$\lim_{p\to+\infty}(\phi(\theta|\overset{o}{Z},R,P))=1-h^{-1}(0)=0. \qquad (69)$$

Finally, since in general $\phi_D(d(A))>0$, (22) implies that the final normalized posterior is

$$\lim_{p\to+\infty}(\phi(\theta|d(A)))=\delta_{\theta,1};\text{all } \theta\in[0,1]. \qquad (70)$$

This may be interpreted as describing the correlation condition "extremely likely correlated", since this can be considered, e.g., a limiting case of the function $\phi_c(\theta)=\theta^q$, for all θ, as $q\to+\infty$, where all intensification is by exponentiation applied to a standard membership function-in this case, the i-

dentity function over [0,1] representing the attribute "correlation level" with domain being probability of correlation or possibility of correlation.

(E) On the other hand, with the conditions in (A) and (B) holding, the lower bound for R is .

$$R(\theta|\mathring{Z}) \geq \min_{(1\leq t\leq r)}(\psi_{\&}(\psi_0(1-G_t(\mathring{Z}),\theta^{\gamma t}),\psi_0(1-\theta^{\gamma t};G_t(\mathring{Z}))))$$
$$\geq \min_{(1\leq t\leq r)}(1-G_t(\mathring{Z})-\theta^{\gamma t})$$
$$\geq \min_{(1\leq t\leq r)}(1-\min_{(k\epsilon J_t)}(M_k(\mathring{Z}_k)^{\beta t,k})-\theta^{\gamma t})$$
$$\overset{d}{=} \rho_0 \,. \quad (71)$$

(F) Thus, if also for $k=1,\ldots,m$

$$\omega \geq M_k(\mathring{Z}_k) \geq 0 \;;\; 0\leq\omega\leq 1 \text{ constant }, \quad (72)$$

then

$$R(\theta|\mathring{Z}) \geq \min_{(1\leq t\leq r)}(1-\omega^{\beta t'}-\theta^{\gamma t})$$
$$\geq 1-\omega^{\beta_0'}-\theta^{\gamma_0'}$$
$$\overset{d}{=} \alpha_0 , \quad (73)$$

where

$$\beta_t' \overset{d}{=} \min_{(k\epsilon J_t)}(\beta_{t,k}) ; \beta_0' \overset{d}{=} \min_{(1\leq t\leq r)}(\beta_t') ; \gamma_0' \overset{d}{=} \min_{(1\leq t\leq r)}(\gamma_t) . \quad (74)$$

Hence, for all $\theta \in [0,1]$,

$$\lim_{p\to+\infty} (\phi(\theta|\mathring{Z},R,P)) \geq 1-h^{-1}(h_{2m},\cdot k(\alpha_0)) . \quad (75)$$

Thus, if also $M_k(\mathring{Z}_k) \to 0$ uniformly, for $k=1,\ldots,m$, β_0' is bounded positively below, and γ' increases without bound. then it follows from (73) and (75) that for all $0\leq\theta<1$,

$$\lim_{p\to+\infty} (\phi(\theta|\mathring{Z},R,P)) \geq 1-h^{-1}(h_{2m},\cdot k(1)) . \quad (76)$$

But, eq.(63) shows the other direction, and hence

$$\lim_{p\to+\infty}(\phi(\theta|\mathring{Z},R,P)) = 1-h^{-1}(h_{2m},\cdot k(1)) > 0 \quad (77)$$

in general (as is true,e.g., for Frank's family[7]). Also note that for $\theta=1$,

$$R(1|\mathring{Z}) = \min_{\substack{1\leq t\leq r,\\ R_t \text{ uses } \Leftrightarrow}} (G_t(\mathring{Z})) \leq \omega^{\beta_0''} , \quad (78)$$

where

$$\beta_0'' \overset{d}{=} \min_{\substack{k\epsilon J_t,\\ 1\leq t\leq r;\ R_t \text{ uses } \Leftrightarrow}}(\beta_{t,k}) . \quad (79)$$

Thus, if there is an R_t using $\Leftrightarrow$, it follows that if β_0'' is bounded positively from below, then the above conditions imply that

$$\lim_{p\to+\infty} (\phi(1|\mathring{Z},R,P)) = 1-h^{-1}(0)=0 , \quad (80)$$

and hence from (22) it follows that for all $\theta \in [0,1]$,

$$\lim_{p\to+\infty} (\phi(\theta|d(A))) = 1 - \delta_{\theta,1} , \quad (81)$$

which,analoguous to the result in(D), indicates linguistically that the condition "not extremely correlated" holds, corresponding to the limiting case for possibility function $\phi_b(\theta)=1-\theta^q$ as a function of θ, as $q\to+\infty$.

In summary, the above results show that sufficiently good information and matching of data (as in (D)) leads to asymptotic consistent results, while relatively poor matching leads to the undesirable results of (F).

6. Concluding Remarks

This paper has shown how a combination of evidence procedure may be established with a general application to the data association problem. Some asymptotic consistency properties of the procedure were established. Implementation of the procedure is necessarily deeply involved with the use of Kalman filters for updating geolocation state vectors, as well as other real-world procedures, including sensor system models and schemes for the clustering of ordered pairs based upon their level of correlation, often presented in the form of a "correlation matrix". The latter depends upon a single figure-of-merit representing the average correlation level (or probability) between any two track histories i,j. One suggestion for this has been to use, in effect, the two stage r.v. $(W|S_U(C))$, where $S_U(C)$ is the naturally corresponding random set(discussed before -see section 3) for output $\phi_C = \phi(\cdot|d(A))$, a function of $\theta\epsilon[0,1]$, as obtained above, and where W in its conditional form is a r.v. uniformly distributed over the outcome $S_U(C)$ [14]. In particular, if ϕ_C is unimodal, then the measure of mean is

$$E(W|S_U(C)) = E(1/2)(\underbrace{\phi_C^{-1}(U)}_{\text{left branch}} + \underbrace{\phi_C^{-1}(U)}_{\text{right branch}}))$$
$$=1/2 (\int_{x=0}^{\mu} x d\phi_C(x) + \int_{x=\mu}^{1} x d(1-\phi_C(x)))$$
$$=\mu\phi_C(\mu) + (1/2)(\int_{x=\mu}^{1}\phi_C(x)dx - \int_{x=0}^{\mu}\phi_C(x)dx -\phi_C(1)), \quad (82)$$

where μ is the mode. An open question connected with this value is the determination of a related figure-of-merit which is invariant with respect to any particular random set,one point coverage equivalent to ϕ_C .

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